OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Evaluate $\sum_{r=101}^{250} r^3$. **[3]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. **[4]**

3 The complex numbers z and w are given by $z = 5 - 2\mathrm{i}$ and $w = 3 + 7\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. **[4]**

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[2]**

6 The complex number $3 - 3\mathrm{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, **[3]**

(b) $\arg(z - a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geqslant 3\sqrt{2} \quad \text{and} \quad 0 \leqslant \arg(z - a) \leqslant \tfrac{1}{4}\pi. \qquad \textbf{[3]}$$

7 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^4 - r^4\} = (n+1)^4 - 1. \qquad \textbf{[2]}$$

(ii) Show that $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$. **[2]**

(iii) Hence show that

$$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2. \qquad \textbf{[6]}$$

8 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the image of the unit square under the transformation represented by **C**. **[3]**

The transformation represented by **C** is equivalent to a transformation S followed by another transformation T.

(ii) Given that S is a shear with the y-axis invariant in which the image of the point (1, 1) is (1, 2), write down the matrix that represents S. **[2]**

(iii) Find the matrix that represents transformation T and describe fully the transformation T. **[6]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **A**. **[3]**

(ii) Hence find the values of a for which **A** is singular. **[3]**

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + y + z &= 2a, \\ x + ay + z &= -1, \\ x + y + 2z &= -1, \end{aligned}$$

have any solutions when

(a) $a = 0$,

(b) $a = 1$.

[4]

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

(i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 27$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**

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1.	984390625 – 25502500 = 958888125	B1 M1 A1	 3 **3**	State correct value of S_{250} or S_{100} Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99}) Obtain correct exact answer
2.	$3a + 5b = 1,\ a + 2b = 1$ $a = -3,\ b = 2$	M1 M1 A1 A1	 4 **4**	Obtain a pair of simultaneous equations Attempt to solve Obtain correct answers.
3.	(i) 11 – 29i (ii) 1 + 41i	B1 B1 B1 B1	2 2 **4**	Correct real and imaginary parts Correct real and imaginary parts
4.	Either $p + q = -1, pq = -8$ $\frac{p+q}{pq}$ $-\frac{7}{8}$ Or $\frac{1}{p} + \frac{1}{q} = 8$ $p + q = 1$ $-\frac{7}{8}$ Or $\frac{-1 \pm \sqrt{33}}{2}$ $-\frac{7}{8}$	B1 B1 M1 A1 B1 B1 M1 A1 M1 A1 M1 A1	 4 **4** 	Both values stated or used Correct expression seen Use their values in their expression Obtain correct answer Substitute $x = \frac{1}{u}$ and use new quadratic Correct value stated Use their values in given expression Obtain correct answer Find roots of given quadratic equation Correct values seen Use their values in given expression Obtain correct answer
5.	(i) $u^3 = \{(-)(5u + 7)\}^2$ $u^3 - 25u^2 - 70u - 49 = 0$ (ii) -70	M1 A1 A1 M1 A1 ft	 3 2 **5**	Use given substitution and rearrange Obtain correct expression, or equivalent Obtain correct final answer Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation Obtain correct answer

6.	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or -45^o AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1 B1 ft	3	Circle, centre (3, -3), through O ft for $(\pm 3, \pm 3)$ only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3 **11**	Area above horizontal through a, below (ii) (b) Outside circle
7.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1		Correct $\sum r$ stated $\sum 1 = n$
		M1*		Consider sum of 4 separate terms on RHS
		*DM1		Required sum is LHS – 3 terms
	$(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$	A1		Correct unsimplified expression
	$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$	A1	6 **10**	Obtain given answer correctly
8.	(i)	B1 B1 B1	3	Find coordinates (0, 0) (3, 1) (2, 1) (5, 2) found Accurate diagram sketched
	(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
	(iii) *Either*	B1		Correct inverse for their (ii) stated
	$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	M1		Post multiply **C** by inverse of (ii)
		A1ft		Correct answer found
	Or	M1		Set up 4 equations for elements from correct matrix multiplication
		A2ft		All elements correct, -1 each error
		B1 B1 B1	6 **11**	Shear, x axis invariant or parallel to x-axis eg image of (1, 1) is (3, 1) **SR** allow s.f. 2 or shearing angle of correct angle to appropriate axis

9.	(i) $a\begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$ $2a^2 - 2a$	M1 A1 A1	 3	Correct expansion process shown Obtain correct unsimplified expression Obtain correct answer
	(ii) $a = 0$ or 1	M1 A1ft A1ft	 3	 Equate their det to 0 Obtain correct answers, ft solving a quadratic
	(iii) (a) (b)	B1 B1 B1 B1	 4	Equations consistent, but non unique solutions Correct equations seen & inconsistent, no solutions
			10	
10.	i) $u_2 = 7$ $u_3 = 19$	M1 A1 A1	 3	Attempt to find next 2 terms Obtain correct answers Show given result correctly
	(ii) $u_n = 2(3^{n-1}) + 1$	M1 A1	 2	Expression involving a power of 3 Obtain correct answer
	(iii) $u_{n+1} = 3(2(3^{n-1})+1) - 2$ $u_{n+1} = 2(3^n) + 1$	B1ft M1 A1 A1 B1	 5	Verify result true when $n = 1$ or $n = 2$ Expression for u_{n+1} using recurrence relation Correct unsimplified answer Correct answer in correct form Statement of induction conclusion
			10	